Supplemental Information for

A Combined Activated Sludge Anaerobic Digestion Model (CASADM) to Understand the Role of Anaerobic Sludge Recycling in Wastewater Treatment Plant Performance

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Appendix A.1: Modeling nomenclature

Variable	Description	Units
b	Decay rate	1/t
C	Concentration	M_s/L^3
f_d	Fraction of biodegradable biomass	-
$k_{UAP\text{ or }EPS}$	Formation rate of UAP or EPS	-
k_j	Hydrolysis rate	1/t
$K_{substrate}$	Half-maximum rate concentration	M_s/L^3
$K_{inhibit}$	Inhibition factor	M_s/L^3
Q	Volumetric flow rate	L^3/t
$\hat{q}$	Maximum utilization rate of substrate	$M_s/(M_x\text{-}t)$
V	Volume	L^3
X	Concentration of solids or biomass	M_x/L^3
Y	True yield	M_x/M_s
γ	Conversion factor	-
Subscripts		
a	Ammonium oxidizing bacteria	
Ac	Acetate	
AD	Anaerobic digester	
Amm	Ammonium (NH_4^+-N)	
Ax	Anoxic tank	
BAP	Biomass associated products	
Cl	Clarifier	
Cl-Sl	Flow from clarifier to sludge thickener	
Cl-St	Flow from clarifier to stabilization tank	
COD	Chemical oxygen demand	
Ct	Contact tank	
DO	Dissolved oxygen	
EPS	Extracellular polymeric substances	
f	Fermenters	
h	Heterotrophs	
hyd	Hydrolysis	
in	Influent to the system	
m	Methanogens	
n	Nitrite oxidizing bacteria	

Subscripts	Description	Units
NaN	Nitrate (NO_3^--N)	
NiN	Nitrite (NO_2^--N)	
out	Effluent from the system (from the clarifier)	
P	Single particles of PCOD	
Sl	Sludge thickener	
Sl-AD	Sludge from the sludge thickener	
Sl-super	Supernatant from sludge thickener	
St	Stabilization tank	
UAP	Utilization associated products	
W	Wasting sludge from AD	

Appendix A.2: Modeling approach and mass balance equations

Here, we explain our basic approach to modeling a complex wastewater treatment plant. All mathematical mass balance models are based on conservation of mass in a system:

$$\begin{matrix}\text{Accumulation}\\ \text{rate of mass}\\ \text{within a}\\ \text{system}\end{matrix} = \begin{matrix}\text{Rate of}\\ \text{mass}\\ \text{entering}\\ \text{the system}\end{matrix} - \begin{matrix}\text{Rate of}\\ \text{mass}\\ \text{leaving}\\ \text{the system}\end{matrix} + \begin{matrix}\text{Generation}\\ \text{rate of}\\ \text{mass in the}\\ \text{system}\end{matrix} - \begin{matrix}\text{Loss rate}\\ \text{of mass in}\\ \text{the system}\end{matrix} \tag{A.1}$$

A system that is at steady-state will have an accumulation rate term equal to zero. The first two terms on the RHS of the equation describe advective transfer of mass in and out of the system. The last two terms on the RHS describe the formation or utilization of mass via chemical or biological reactions.

We begin by first identifying the different systems for modeling, which are illustrated in Figure 1 of the main text. We then identify the physical, chemical, and biological mechanisms and solid and soluble components that occur in the system; these are summarized in Table A.1. While the most typical physical mechanism is advective mass transport from tank to tank, separation also occurs in the settlers between a supernatant phase and a sludge phase.

Table A.1. Solid and soluble components in the mathematical model

Solid Components		**Soluble/Gaseous Components**	
		Substrate	Biomass associated products (BAP)
Heterotrophs	Methanogens	NH_4^+	Utilization associated products (UAP)
AOB	PCOD	NO_2^-	Acetate
NOB	Inert biomass	NO_3^-	N_2
Fermenters	EPS	Dissolved oxygen (DO)	CH_4

As stated in the main text, all biomass undergoes three common phenomena: substrate utilization for cell biomass synthesis, endogenous decay and respiration, and formation of SMP and EPS. However, environmental conditions in the activated sludge and anaerobic digestion processes will encourage a variety of other chemical/biological processes:

- Aerobic utilization of acetate and COD by heterotrophs
- Nitrification of NH_4^+ and NO_2^- by AOB and NOB, respectively, under aerobic conditions
- Denitrification of NO_2^- and NO_3^- directly to N_2 by heterotrophs without formation of intermediates in anoxic conditions
- Fermentation of COD to acetate by fermenters in anaerobic conditions
- Production of methane via methanogenesis in anaerobic conditions
- Consumption of SMP and EPS by heterotrophs and fermenters
- Hydrolysis of PCOD and inactive biomass

A discussion of specific mechanisms is in the next section.

Table A.2 summarizes the generic mass balance equations (from Eqn. E.1) developed for each tank and the overall system for the hybrid and conventional processes. Note that the subscripts refer specifically to a tank, and C can refer to concentrations of soluble substrates or biomass.

Table A.2. Mass balance equations for SWT's hybrid and conventional processes, including equations for the overall system and each tank.

Overall system		$V_{\mathrm{system}}\frac{dC}{dt} = Q_{\mathrm{in}}C_{\mathrm{in}} - Q_{\mathrm{out}}C_{\mathrm{out}} - Q_{\mathrm{w}}C_{\mathrm{w}} + R_{\mathrm{system}}$
Anoxic tank (Ax)		$V_{\mathrm{Ax}}\frac{dC_{\mathrm{Ax}}}{dt} = Q_{\mathrm{in}}C_{\mathrm{in}} + Q_{\mathrm{St}}C_{\mathrm{St}} - Q_{\mathrm{Ax}}C_{\mathrm{Ax}} + R_{\mathrm{Ax}}$
Contact tank (Ct)		$V_{\mathrm{Ct}}\frac{dC_{\mathrm{Ct}}}{dt} = Q_{\mathrm{Ax}}C_{\mathrm{Ax}} - Q_{\mathrm{Ct}}C_{\mathrm{Ct}} + R_{\mathrm{Ct}}$
Clarifier (Cl)		$V_{\mathrm{Cl}}\frac{d(C_{\mathrm{Cl}} + C_{\mathrm{out}})}{dt} = Q_{\mathrm{Ct}}C_{\mathrm{Ct}} - Q_{\mathrm{out}}C_{\mathrm{out}} - Q_{\mathrm{Cl}}C_{\mathrm{Cl}} + R_{\mathrm{Cl}}$
Sludge thickener (Sl)		$V_{\mathrm{Sl}}\frac{dC_{\mathrm{Sl}}}{dt} = Q_{\mathrm{Cl-Sl}}C_{\mathrm{Cl}} - Q_{\mathrm{Sl-AD}}C_{\mathrm{Sl-AD}} - Q_{\mathrm{Sl-super}}C_{\mathrm{Sl-super}} + R_{\mathrm{Sl}}$ where $C_{\mathrm{Sl}} = \frac{C_{\mathrm{Sl-super}}Q_{\mathrm{Sl-super}} + C_{\mathrm{Sl-AD}}Q_{\mathrm{Sl-AD}}}{Q_{\mathrm{Sl-super}} + Q_{\mathrm{Sl-AD}}}$
Stabilization tank (St)	Hybrid	$V_{\mathrm{St}}\frac{dC_{\mathrm{St}}}{dt} = Q_{\mathrm{Cl-St}}C_{\mathrm{Cl}} + Q_{\mathrm{Sl-St}}C_{\mathrm{Sl}} + Q_{\mathrm{AD}}C_{\mathrm{W}} - Q_{\mathrm{St}}C_{\mathrm{St}} + R_{\mathrm{St}}$
	Conventional	$V_{\mathrm{St}}\frac{dC_{\mathrm{St}}}{dt} = Q_{\mathrm{Cl-St}}C_{\mathrm{Cl}} + Q_{\mathrm{Sl-St}}C_{\mathrm{Sl}} - Q_{\mathrm{St}}C_{\mathrm{St}} + R_{\mathrm{St}}$
Anaerobic digester (AD)	Hybrid	$V_{\mathrm{AD}}\frac{dC_{\mathrm{AD}}}{dt} = Q_{\mathrm{Sl-AD}}C_{\mathrm{Sl}} - C_{\mathrm{W}}(Q_{\mathrm{W}} + Q_{\mathrm{AD}}) + R_{\mathrm{AD}}$
	Conventional	$V_{\mathrm{AD}}\frac{dC_{\mathrm{AD}}}{dt} = Q_{\mathrm{Sl-AD}}C_{\mathrm{Sl}} - C_{\mathrm{W}}Q_{\mathrm{W}} + R_{\mathrm{AD}}$

Appendix A.3: Model Features

A.3.1. Dual-limitation Monod kinetics

As established in Bae and Rittmann (1996), dual-limitation Monod kinetics are applied to describe situations in which the reaction rate is limited by the electron-donor concentration, electron-acceptor concentration, or both. For example, when COD is aerobically oxidized by heterotrophs, the reaction rate, r, is described as

$$\mathrm{r} = \hat{\mathrm{q}}\left(\frac{\hat{\mathrm{q}}_{\mathrm{S}}\mathrm{S}}{\mathrm{K}_{\mathrm{S}} + \mathrm{S}}\right)\left(\frac{\mathrm{DO}}{\mathrm{K}_{\mathrm{DO}} + \mathrm{DO}}\right)\mathrm{X}_{\mathrm{h}} \qquad (\mathrm{A.2})$$

where $\hat{q}$ is the maximum utilization rate of substrate (M_s/M_x-t), S and DO are COD substrate and dissolved oxygen (DO) concentrations (M_s/L^3), respectively, K_S and K_{DO} are the substrate and DO half-maximum-rate concentrations (M_s/L^3), respectively, and X_h is the concentration of heterotrophs (M_x/L^3). Thus, if the electron donor or substrate concentrations are below saturation, the Monod term decreases and, under extreme limitation, becomes very small.

A.3.2. Inhibition (or switch) factors

To simplify the model implementation, we assumed that any mechanism could occur in any tank. Based on de Silva and Rittmann (2000), the level of activity of any mechanism is controlled through the application of an inhibition or switch factor. Different mechanisms described in this model can undergo inhibition if DO, nitrate (NO_3^-), and/or nitrite (NO_2^-) are present. The inhibition factor for DO is

$$\mathrm{DO}_{\mathrm{switch}} = \frac{\mathrm{K}_{\mathrm{s,DO}}}{\mathrm{K}_{\mathrm{s,DO}} + \mathrm{DO}} \qquad (\mathrm{A.3})$$

where $K_{s,DO}$ is the inhibition factor for DO (M/L^3). When the DO concentration is low, DO^{switch} is ~1, turning the switch on to describe processes under low DO concentrations. When the DO concentration is low, DO^{switch} is ~0, turning the switch off to inhibit processes under aerobic conditions. Similarly, the switch factors for NO_2^-, $NO_{2,switch}$, and NO_3^-, $NO_{3,switch}$, are:

$$\mathrm{NO_{2,switch}} = \frac{\mathrm{K_{s,NO_2}}}{\mathrm{K_{s,NO_2}} + \mathrm{NO_2}} \qquad \text{(A. 4)}$$

$$\mathrm{NO_{3,switch}} = \frac{\mathrm{K_{s,NO_3}}}{\mathrm{K_{s,NO_3}} + \mathrm{NO_3}} \qquad \text{(A. 5)}$$

where K_{s,NO_2} and K_{s,NO_3} are the inhibition factors of NO_2^- and NO_3^- (M/L^3), respectively, and NO_2 and NO_3 are the concentrations of NO_2^- and NO_3^- in the tank (M/L^3), respectively. NO_2^- and NO_3^- switches are ~1, the switches are turned on.

These switches can be applied multiplicatively to describe several situations at once. For example, anaerobic conditions can be activated when the following switch expression is multiplied to an anaerobic rate equation:

$$\mathrm{DO_{switch}}\left(\mathrm{NO_{2,switch}} + \mathrm{NO_{3,switch}}\right) \qquad \text{(A. 6)}$$

The NO_x switches are added together as denitrification is dependent upon the total amount of NO_2^- and NO_3^- in the system.

A.3.4. Application of the Unified Theory of EPS and SMP

Laspidou and Rittmann (2002a, 2002b) outline the unified theory of EPS and SMP in their fundamental work. We modified their theory to expand its application to the variety of mechanisms presented in this paper. Consistent with Laspidou and Rittmann (2002a), all microorganisms produce EPS and UAP. Biomass yield is reduced as electrons are diverted to EPS and UAP formation by a factor of $1\text{-}k_{UAP}\text{-}k_{EPS}$, where k_{UAP} represents the fraction of electrons going to UAP formation (M_s/M_s) and k_{EPS} represents the fraction of electrons going to EPS formation (M_s/M_s). The factor $1\text{-}k_{UAP}\text{-}k_{EPS}$ is represented as “c” in Table A.3. EPS is hydrolyzed to BAP using first-order kinetics. BAP can then be consumed by microorganisms for growth.

Only heterotrophs and fermenters utilize UAP and BAP, as they are heterotrophic, COD-consuming microorganisms. UAP and BAP consumption follows Monod-based substrate utilization kinetics. When UAP and BAP are utilized, microorganisms convert the energy to biomass based on a yield, Y_p (M_x/M_s), which is assumed to be different than the direct utilization of other substrates. The microorganisms can produce additional EPS and SMP from utilization of UAP and BAP.

A.3.4. Stoichiometric coefficients

The half reaction for cell synthesis is (Rittmann & McCarty, 2001):

$$\frac{1}{2}\mathrm{CO_2} + \frac{1}{20}\mathrm{NH_4^+} + \frac{1}{20}\mathrm{HCO_3^-} + \mathrm{H^+} + \mathrm{e^-} = \frac{1}{20}\mathrm{C_5H_7O_2N} + \frac{9}{20}\mathrm{H_2O} \qquad (\mathrm{A.7})$$

where the biomass molecular formula is $C_5H_7O_2N$ (Rittmann and McCarty, 2001; Metcalf & Eddy, Inc., 2003). Thus, the conversion of cell biomass to COD, γ_o, can be determined by equating the substrates to their electron equivalents:

$$\gamma_\mathrm{O} = \frac{1\ \text{mol cells}}{113\text{g cells}} * \frac{20\ \mathrm{e^-}\ \text{eq}}{1\ \text{mol cells}} * \frac{8\text{g COD}}{1\mathrm{e^-}\ \text{eq}} = \frac{160\ \text{g COD}}{113\ \text{g cells}} \qquad (\mathrm{A.8})$$

Similarly, the amount of nitrogen in cells is

$$\gamma_\mathrm{N} = \frac{1\ \text{mol cells}}{113\text{g cells}} * \frac{1\ \text{mol N}}{1\ \text{mol cells}} * \frac{14\text{g N}}{1\ \text{mol N}} = \frac{14\ \text{g N}}{113\ \text{g cells}} \qquad (\mathrm{A.9})$$

Similar conversions can be performed for other substrates based on electron equivalents. For denitrification of NO_3^- and NO_2^- to N_2 gas, the half reactions are

$$\frac{1}{5}\mathrm{NO_3^-} + \frac{6}{5}\mathrm{H^+} + \mathrm{e^-} = \frac{1}{10}\mathrm{N_2} + \frac{3}{5}\mathrm{H_2O} \qquad (\mathrm{A.10})$$

$$\frac{1}{3}\mathrm{NO_2^-} + \frac{4}{3}\mathrm{H^+} + \mathrm{e^-} = \frac{1}{6}\mathrm{N_2} + \frac{2}{3}\mathrm{H_2O} \qquad (\mathrm{A.11})$$

To convert NO_3^- (γ_{NaN}) and NO_2^- (γ_{NiN}) to mgN/mgCOD,

$$\gamma_{\mathrm{NaN}} = \frac{14\ \mathrm{gN}}{1\ \mathrm{mol\ NO_3^-}} * \frac{1\ \mathrm{mol\ NO_3^-}}{5\mathrm{e^-eq}} * \frac{1\mathrm{e^-\ eq}}{8\mathrm{g\ COD}} = \frac{14\ \mathrm{g\ N}}{40\ \mathrm{g\ COD}} \tag{A.12}$$

$$\gamma_{\mathrm{NiN}} = \frac{14\ \mathrm{gN}}{1\ \mathrm{mol\ NO_2^-}} * \frac{1\ \mathrm{mol\ NO_2^-}}{3\mathrm{e^-eq}} * \frac{1\mathrm{e^-\ eq}}{8\mathrm{g\ COD}} = \frac{14\ \mathrm{g\ N}}{24\ \mathrm{g\ COD}} \tag{A.13}$$

For acetate, the half reaction is

$$\frac{1}{8}\mathrm{CO_2} + \frac{1}{8}\mathrm{HCO_3^-} + \mathrm{H^+} + \mathrm{e^-} = \frac{1}{8}\mathrm{CH_3COO^-} + \frac{3}{8}\mathrm{H_2O} \tag{A.14}$$

To convert acetate to COD, γ_A,

$$\gamma_{\mathrm{A}} = \frac{1\ \mathrm{mol\ acetate}}{60\ \mathrm{g\ acetate}} * \frac{8\mathrm{e^-\ eq}}{1\ \mathrm{mol\ acetate}} * \frac{8\mathrm{g\ COD}}{1\mathrm{e^-\ eq}} = \frac{64\ \mathrm{g\ COD}}{60\ \mathrm{g\ acetate}} \tag{A.15}$$

A critical part of the DO calculations is equating the DO utilized by AOB and NOB to the appropriate number of electron equivalents utilized. During nitrification, AOB convert NH_4^+ to NO_2^- via the half reaction

$$\frac{1}{6}\mathrm{NO_2^-} + \frac{4}{3}\mathrm{H^+} + \mathrm{e^-} = \frac{1}{6}\mathrm{NH_4^+} + \frac{1}{3}\mathrm{H_2O} \tag{A.16}$$

To convert NH_4^+ to NO_2^- and express it as COD,

$$\gamma_1 = \frac{\mathrm{mol\ NH_4}}{14\ \mathrm{g\ N}} * \frac{6\ \mathrm{e^-\ eq}}{1\ \mathrm{mol\ NH_4}} * \frac{8\ \mathrm{g\ COD}}{\mathrm{e^-\ eq}}\frac{48\ \mathrm{g\ COD}}{14\ \mathrm{g\ N}} = 3.43\frac{\mathrm{g\ COD}}{\mathrm{g\ N}} \tag{A.17}$$

NOB convert NO_2^- to NO_3^-, the half reaction is

$$\frac{1}{2}\mathrm{NO_3^-} + \mathrm{H^+} + \mathrm{e^-} = \frac{1}{2}\mathrm{NO_2^-} + \frac{1}{2}\mathrm{H_2O} \tag{A.18}$$

To convert NO_2^- to NO_3^- and express it as COD,

$$\gamma_1 = \frac{\mathrm{mol\ NH_4}}{14\ \mathrm{g\ N}} * \frac{2\ \mathrm{e^-\ eq}}{1\ \mathrm{mol\ NH_4}} * \frac{8\ \mathrm{g\ COD}}{\mathrm{e^-\ eq}} = \frac{16\ \mathrm{g\ COD}}{14\ \mathrm{g\ N}} = 1.14\frac{\mathrm{g\ COD}}{\mathrm{g\ N}} \tag{A.19}$$

Table A.3. Process, stoichiometry, and kinetics matrix. A blank cell indicates 0.

	Process	Chemical components					Kinetic expression
		Substrate S	PCOD	Acetate	UAP	BAP	
1	Aerobic metabolism of substrate by X_h	-1			k_{UAP}		$\left(\frac{\hat{q}_S S}{K_{S,h}+S}\right)\left(\frac{DO}{K_{DO,h}+DO}\right)X_h$
2	Anoxic metabolism of substrate by X_h with NO_3^{-}N	-1			k_{UAP}		$a_3\left(\frac{\hat{q}_{Nan} S}{K_{S,h}+S}\right)\left(\frac{NaN}{K_{NaN,h}+NaN}\right)X_h$
3	Anoxic metabolism of substrate by X_h with NO_2^{-}N	-1			k_{UAP}		$a_3\left(\frac{\hat{q}_{Nin} S}{K_{S,h}+S}\right)\left(\frac{NiN}{K_{NiN,h}+NiN}\right)X_h$
4	Hydrolysis of inactive biomass	$\gamma_O(X_h+X_a+X_n)$					$a_3(a_1+a_2)k_{hyd}$
5	Anaerobic utilization of substrate by X_f	-1		$c(1-\gamma_O Y_f)$	k_{UAP}		$a_3(a_1+a_2)\left(\frac{\hat{q}_S S}{K_{S,f}+S}\right)X_f$
6	Aerobic metabolism of NO_2^--N by X_n				$\frac{\gamma_O}{\gamma_N}k_{UAP}$		$\left(\frac{\hat{q}_n NiN}{K_{NiN,n}+NiN}\right)\left(\frac{DO}{K_{DO,n}+DO}\right)X_n$
7	Aerobic metabolism of NH_4^+-N by X_a				$\frac{\gamma_O}{\gamma_N}k_{UAP}$		$\left(\frac{\hat{q}_a Amm}{K_{Amm}+Amm}\right)\left(\frac{DO}{K_{DO,a}+DO}\right)X_a$
8	Aerobic metabolism of acetate by X_h			-1	k_{UAP}		$\left(\frac{\hat{q}_{Ac} Ac}{K_{Ac,h}+Ac}\right)\left(\frac{DO}{K_{DO,h}+DO}\right)X_h$
9	Anoxic metabolism of acetate by X_h with NO_3^{-}N			-1	k_{UAP}		$a_3\left(\frac{\hat{q}_{Ac} Ac}{K_{Ac,h}+Ac}\right)\left(\frac{NaN}{K_{NaN,h}+NaN}\right)X_h$

		Chemical components					
	Process	Substrate S	PCOD	Acetate	UAP	BAP	Kinetic expression
10	Anoxic metabolism of acetate by X_h with NO_2^{-}-N			-1	k_{UAP}		$a_3\left(\frac{\hat{q}_{Ac}Ac}{K_{Ac,h}+Ac}\right)\left(\frac{NiN}{K_{NiN,h}+NiN}\right)X_h$
11	Acetate utilization by X_m			-1	k_{UAP}		$a_3(a_1+a_2)\left(\frac{\hat{q}_{Ac,m}Ac}{K_{Ac,m}+Ac}\right)X_m$
12	Aerobic metabolism of UAP by X_h				$-(1-k_{UAP})$		$\left(\frac{\hat{q}_{UAP,h}UAP}{UAP+K_{UAP,h}}\right)\left(\frac{DO}{K_{DO,h}+DO}\right)X_h$
13	Anoxic metabolism of UAP and NO_3^{-}-N by X_h				$-(1-k_{UAP})$		$a_3\left(\frac{\hat{q}_{UAP,h}UAP}{UAP+K_{UAP,h}}\right)\left(\frac{NaN}{K_{NaN,h}+NaN}\right)X_h$
14	Anoxic metabolism of UAP and NO_2^{-}-N by X_h				$-(1-k_{UAP})$		$a_3\left(\frac{\hat{q}_{UAP,h}UAP}{UAP+K_{UAP,h}}\right)\left(\frac{NiN}{K_{NiN,h}+NiN}\right)X_h$
15	Metabolism of UAP by X_f			$c(1-\gamma_O Y_f)$	$-(1-k_{UAP})$		$a_3(a_1+a_2)\left(\frac{\hat{q}_{UAP,f}UAP}{UAP+K_{UAP,f}}\right)X_f$
16	Aerobic metabolism of BAP by X_h				k_{UAP}	-1	$\left(\frac{\hat{q}_{BAP,h}BAP}{BAP+K_{BAP,h}}\right)\left(\frac{DO}{K_{DO,h}+DO}\right)X_h$
17	Anoxic metabolism of BAP and NO_3^{-}-N by X_h				k_{UAP}	-1	$a_3\left(\frac{\hat{q}_{BAP,h}BAP}{BAP+K_{BAP,h}}\right)\left(\frac{NaN}{K_{NaN,h}+NaN}\right)X_h$
18	Anoxic metabolism of BAP and NO_2^{-}-N by X_h				k_{UAP}	-1	$a_3\left(\frac{\hat{q}_{BAP,h}BAP}{BAP+K_{BAP,h}}\right)\left(\frac{NiN}{K_{NiN,h}+NiN}\right)X_h$
19	Metabolism of BAP by X_f			$c(1-\gamma_O Y_f)$	k_{UAP}	-1	$a_3(a_1+a_2)\left(\frac{\hat{q}_{BAP,f}BAP}{BAP+K_{BAP,f}}\right)X_f$
20	Hydrolysis of PCOD	1	-1				$k_P P$

	Process	Chemical components					Kinetic expression
		Substrate S	PCOD	Acetate	UAP	BAP	
21	Aerobic respiration by X_h						$f_d b_h \left(\frac{DO}{K_{DO,h} + DO}\right) X_h$
22	Anoxic respiration by X_h during NO_3^--N utilization						$f_d b_h a_3 \left(\frac{NiN}{K_{NiN,h} + NiN}\right) X_h$
23	Anoxic respiration by X_h during NO_2^--N utilization						$f_d b_h a_3 \left(\frac{NaN}{K_{NaN,h} + NaN}\right) X_h$
24	Aerobic respiration by X_a						$f_d b_a \left(\frac{DO}{K_{DO,a} + DO}\right) X_a$
25	Aerobic respiration by X_n						$f_d b_n \left(\frac{DO}{K_{DO,n} + DO}\right) X_n$
26	Anaerobic respiration of X_f			1			$f_d b_f X_f a_3 (a_1 + a_2)$
27	Anaerobic respiration of X_m						$f_d b_m X_m a_3 (a_1 + a_2)$
28	Formation of BAP					1	$k_{hyd}EPS$
29	Decay of X_h						$b_h X_h \left[\frac{DO}{K_{DO,h} + DO} + \frac{NaN}{K_{NaN,h} + NaN} + \frac{NiN}{K_{NiN,h} + NiN}\right]$
30	Decay of X_a						$b_a \left(\frac{DO}{K_{DO,a} + DO}\right) X_a$
31	Decay of X_n						$b_n \left(\frac{DO}{K_{DO,n} + DO}\right) X_n$
32	Decay of X_f						$\gamma_O b_f X_f a_3 (a_1 + a_2)$

	Process	Chemical components					Kinetic expression
		Substrate S	PCOD	Acetate	UAP	BAP	
33	Decay of X_m						$\gamma_O b_m X_m a_3 (a_1 + a_2)$
34	UNITS	$\frac{mgCOD}{L\ d}$	$\frac{mgCOD}{L\ d}$	$\frac{mgCOD}{L\ d}$	$\frac{mgCOD}{L\ d}$	$\frac{mgCOD}{L\ d}$	

	Process	Chemical components					Kinetic expression
		EPS	DO	NH_4^+-N	NO_3^--N	NO_2^--N	
1	Aerobic metabolism of substrate by X_h	$\frac{k_{EPS}}{\gamma_O}$	$-\alpha_h$	$-\left(\gamma_N c Y_h + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$			$\left(\frac{\hat{q}_S S}{K_{S,h} + S}\right)\left(\frac{DO}{K_{DO,h} + DO}\right)X_h$
2	Anoxic metabolism of substrate by X_h with NO_3^--N	$\frac{k_{EPS}}{\gamma_O}$		$-\left(\gamma_N c Y_h + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$	$-\gamma_{NaN}$		$a_3\left(\frac{\hat{q}_{Nan} S}{K_{S,h} + S}\right)\left(\frac{NaN}{K_{NaN,h} + NaN}\right)X_h$
3	Anoxic metabolism of substrate by X_h with NO_2^--N	$\frac{k_{EPS}}{\gamma_O}$		$-\left(\gamma_N c Y_h + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$		$-\gamma_{NiN}$	$a_3\left(\frac{\hat{q}_{Nin} S}{K_{S,h} + S}\right)\left(\frac{NiN}{K_{NiN,h} + NiN}\right)X_h$
4	Hydrolysis of inactive biomass			γ_N			$a_3(a_1 + a_2)k_{hyd}$
5	Anaerobic utilization of substrate by X_f	$\frac{k_{EPS}}{\gamma_O}$		$-\left(\gamma_N c Y_f + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$			$a_3(a_1 + a_2)\left(\frac{\hat{q}_S S}{K_{S,f} + S}\right)X_f$
6	Aerobic metabolism of NO_2^--N by X_n	$\frac{k_{EPS}}{\gamma_N}$	$-\alpha_n$	k_{UAP}	$c(1-Y_n\gamma_N)$	-1	$\left(\frac{\hat{q}_n NiN}{K_{NiN,n} + NiN}\right)\left(\frac{DO}{K_{DO,n} + DO}\right)X_n$
7	Aerobic metabolism of NH_4^+-N by X_a	$\frac{k_{EPS}}{\gamma_N}$	$-\alpha_a$	$-(1- k_{UAP})$		$c(1-Y_a\gamma_N)$	$\left(\frac{\hat{q}_a Amm}{K_{Amm} + Amm}\right)\left(\frac{DO}{K_{DO,a} + DO}\right)X_a$
8	Aerobic metabolism of acetate by X_h	$\frac{k_{EPS}}{\gamma_O}$	$-\alpha_h$	$-\left(\gamma_N c Y_h + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$			$\left(\frac{\hat{q}_{Ac} Ac}{K_{Ac,h} + Ac}\right)\left(\frac{DO}{K_{DO,h} + DO}\right)X_h$

	Process	Chemical components					Kinetic expression
		EPS	DO	NH_4^+-N	NO_3^--N	NO_2^--N	
9	Anoxic metabolism of acetate by X_h with NO_3^--N	$\frac{k_{EPS}}{\gamma_O}$		$-\left(\gamma_N c Y_h + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$	$-\gamma_{NaN}$		$a_3\left(\frac{\hat{q}_{Ac}Ac}{K_{Ac,h} + Ac}\right)\left(\frac{NaN}{K_{NaN,h} + NaN}\right)X_h$
10	Anoxic metabolism of acetate by X_h with NO_2^--N	$\frac{k_{EPS}}{\gamma_O}$		$-\left(\gamma_N c Y_h + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$		$-\gamma_{NiN}$	$a_3\left(\frac{\hat{q}_{Ac}Ac}{K_{Ac,h} + Ac}\right)\left(\frac{NiN}{K_{NiN,h} + NiN}\right)X_h$
11	Acetate utilization by X_m	$\frac{k_{EPS}}{\gamma_O}$		$-\left(\gamma_N c Y_m + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$			$a_3(a_1 + a_2)\left(\frac{\hat{q}_{Ac,m}Ac}{K_{Ac,m} + Ac}\right)X_m$
12	Aerobic metabolism of UAP by X_h	$\frac{k_{EPS}}{\gamma_O}$	$-\alpha_{hp}$	$-\left(\gamma_N c Y_p + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$			$\left(\frac{\hat{q}_{UAP,h}UAP}{UAP + K_{UAP,h}}\right)\left(\frac{DO}{K_{DO,h} + DO}\right)X_h$
13	Anoxic metabolism of UAP and NO_3^--N by X_h	$\frac{k_{EPS}}{\gamma_O}$		$-\left(\gamma_N c Y_p + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$	$-\gamma_{NaN}$		$a_3\left(\frac{\hat{q}_{UAP,h}UAP}{UAP + K_{UAP,h}}\right)\left(\frac{NaN}{K_{NaN,h} + NaN}\right)X_h$
14	Anoxic metabolism of UAP and NO_2^--N by X_h	$\frac{k_{EPS}}{\gamma_O}$		$-\left(\gamma_N c Y_p + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$		$-\gamma_{NiN}$	$a_3\left(\frac{\hat{q}_{UAP,h}UAP}{UAP + K_{UAP,h}}\right)\left(\frac{NiN}{K_{NiN,h} + NiN}\right)X_h$
15	Metabolism of UAP by X_f	$\frac{k_{EPS}}{\gamma_O}$		$-\left(\gamma_N c Y_p + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$			$a_3(a_1 + a_2)\left(\frac{\hat{q}_{UAP,f}UAP}{UAP + K_{UAP,f}}\right)X_f$

	Process	Chemical components					Kinetic expression
		EPS	DO	NH_4^+-N	NO_3^--N	NO_2^--N	
16	Aerobic metabolism of BAP by X_h	$\frac{k_{EPS}}{\gamma_O}$	$-\alpha_{hp}$	$-\left(\gamma_N c Y_p + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$			$\left(\frac{\hat{q}_{BAP,h} BAP}{BAP + K_{BAP,h}}\right)\left(\frac{DO}{K_{DO,h} + DO}\right) X_h$
17	Anoxic metabolism of BAP and NO_3^--N by X_h	$\frac{k_{EPS}}{\gamma_O}$		$-\left(\gamma_N c Y_p + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$	$-\gamma_{NaN}$		$a_3\left(\frac{\hat{q}_{BAP,h} BAP}{BAP + K_{BAP,h}}\right)\left(\frac{NaN}{K_{NaN,h} + NaN}\right) X_h$
18	Anoxic metabolism of BAP and NO_2^--N by X_h	$\frac{k_{EPS}}{\gamma_O}$		$-\left(\gamma_N c Y_p + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$		$-\gamma_{NiN}$	$a_3\left(\frac{\hat{q}_{BAP,h} BAP}{BAP + K_{BAP,h}}\right)\left(\frac{NiN}{K_{NiN,h} + NiN}\right) X_h$
19	Metabolism of BAP by X_f	$\frac{k_{EPS}}{\gamma_O}$		$-\left(\gamma_N c Y_f + \frac{k_{EPS}\gamma_N}{\gamma_O}\right)$			$a_3(a_1 + a_2)\left(\frac{\hat{q}_{BAP,f} BAP}{BAP + K_{BAP,f}}\right) X_f$
20	Hydrolysis of PCOD						$k_P P$
21	Aerobic respiration by X_h		$-\gamma_O$	γ_N			$f_d b_h \left(\frac{DO}{K_{DO,h} + DO}\right) X_h$
22	Anoxic respiration by X_h during NO_3^--N utilization			γ_N	$-\gamma_{NaN}\gamma_O$		$f_d b_h a_3 \left(\frac{NiN}{K_{NiN,h} + NiN}\right) X_h$
23	Anoxic respiration by X_h during NO_2^--N utilization			γ_N		$-\gamma_{NiN}\gamma_O$	$f_d b_h a_3 \left(\frac{NaN}{K_{NaN,h} + NaN}\right) X_h$
24	Aerobic respiration by X_a		$-\gamma_O$	γ_N			$f_d b_a \left(\frac{DO}{K_{DO,a} + DO}\right) X_a$
25	Aerobic respiration by X_n		$-\gamma_O$	γ_N			$f_d b_n \left(\frac{DO}{K_{DO,n} + DO}\right) X_n$

	Process	Chemical components					Kinetic expression
		EPS	DO	NH_4^+-N	NO_3^--N	NO_2^--N	
26	Anaerobic respiration of X_f			γ_N			$\gamma_O f_d b_f X_f a_3 (a_1 + a_2)$
27	Anaerobic respiration of X_m			γ_N			$\gamma_O f_d b_m X_m a_3 (a_1 + a_2)$
28	Formation of BAP	-1		γ_N			$k_{hyd}EPS$
29	Decay of X_h						$b_h X_h \left[\frac{DO}{K_{DO,h} + DO} + \frac{NaN}{K_{NaN,h} + NaN} + \frac{NiN}{K_{NiN,h} + NiN}\right]$
30	Decay of X_a						$b_a \left(\frac{DO}{K_{DO,a} + DO}\right) X_a$
31	Decay of X_n						$b_n \left(\frac{DO}{K_{DO,n} + DO}\right) X_n$
32	Decay of X_f						$b_f X_f a_3 (a_1 + a_2)$
33	Decay of X_m						$b_m X_m a_3 (a_1 + a_2)$
34	UNITS	$\frac{mgVSS}{L\ d}$	$\frac{mgCOD}{L\ d}$	$\frac{mgNH_4^+ - N}{L\ d}$	$\frac{mgNO_3^- - N}{L\ d}$	$\frac{mgNO_2^- - N}{L\ d}$	

	Process	Chemical components		Biomass components			Kinetic expression
		N_2	CH_4	X_h	X_a	X_n	
1	Aerobic metabolism of substrate by X_h			cY_h			$\left(\frac{\hat{q}_S S}{K_{S,h}+S}\right)\left(\frac{DO}{K_{DO,h}+DO}\right)X_h$
2	Anoxic metabolism of substrate by X_h with NO_3^--N	γ_{NaN}		cY_h			$a_3\left(\frac{\hat{q}_{Nan} S}{K_{S,h}+S}\right)\left(\frac{NaN}{K_{NaN,h}+NaN}\right)X_h$
3	Anoxic metabolism of substrate by X_h with NO_2^--N	γ_{NiN}		cY_h			$a_3\left(\frac{\hat{q}_{Nin} S}{K_{S,h}+S}\right)\left(\frac{NiN}{K_{NiN,h}+NiN}\right)X_h$
4	Hydrolysis of inactive biomass			-1	-1	-1	$a_3(a_1+a_2)k_{hyd}$
5	Anaerobic utilization of substrate by X_f						$a_3(a_1+a_2)\left(\frac{\hat{q}_S S}{K_{S,f}+S}\right)X_f$
6	Aerobic metabolism of NO_2^--N by X_n					cY_n	$\left(\frac{\hat{q}_n NiN}{K_{NiN,n}+NiN}\right)\left(\frac{DO}{K_{DO,n}+DO}\right)X_n$
7	Aerobic metabolism of NH_4^+-N by X_a				cY_a		$\left(\frac{\hat{q}_a Amm}{K_{Amm}+Amm}\right)\left(\frac{DO}{K_{DO,a}+DO}\right)X_a$
8	Aerobic metabolism of acetate by X_h			cY_h			$\left(\frac{\hat{q}_{Ac} Ac}{K_{Ac,h}+Ac}\right)\left(\frac{DO}{K_{DO,h}+DO}\right)X_h$
9	Anoxic metabolism of acetate by X_h with NO_3^--N	γ_{NaN}		cY_h			$a_3\left(\frac{\hat{q}_{Ac} Ac}{K_{Ac,h}+Ac}\right)\left(\frac{NaN}{K_{NaN,h}+NaN}\right)X_h$

	Process	Chemical components		Biomass components			Kinetic expression
		N_2	CH_4	X_h	X_a	X_n	
10	Anoxic metabolism of acetate by X_h with NO_2^--N	γ_{NiN}		cY_h			$a_3\left(\frac{\hat{q}_{Ac}Ac}{K_{Ac,h}+Ac}\right)\left(\frac{NiN}{K_{NiN,h}+NiN}\right)X_h$
11	Acetate utilization by X_m		$c(1-Y_m\gamma_O)$				$a_3(a_1+a_2)\left(\frac{\hat{q}_{Ac,m}Ac}{K_{Ac,m}+Ac}\right)X_m$
12	Aerobic metabolism of UAP by X_h			cY_p			$\left(\frac{\hat{q}_{UAP,h}UAP}{UAP+K_{UAP,h}}\right)\left(\frac{DO}{K_{DO,h}+DO}\right)X_h$
13	Anoxic metabolism of UAP and NO_3^--N by X_h	γ_{NaN}		cY_p			$a_3\left(\frac{\hat{q}_{UAP,h}UAP}{UAP+K_{UAP,h}}\right)\left(\frac{NaN}{K_{NaN,h}+NaN}\right)X_h$
14	Anoxic metabolism of UAP and NO_2^--N by X_h	γ_{NiN}		cY_p			$a_3\left(\frac{\hat{q}_{UAP,h}UAP}{UAP+K_{UAP,h}}\right)\left(\frac{NiN}{K_{NiN,h}+NiN}\right)X_h$
15	Metabolism of UAP by X_f						$a_3(a_1+a_2)\left(\frac{\hat{q}_{UAP,f}UAP}{UAP+K_{UAP,f}}\right)X_f$
16	Aerobic metabolism of BAP by X_h			cY_p			$\left(\frac{\hat{q}_{BAP,h}BAP}{BAP+K_{BAP,h}}\right)\left(\frac{DO}{K_{DO,h}+DO}\right)X_h$
17	Anoxic metabolism of BAP and NO_3^--N by X_h	γ_{NaN}		cY_p			$a_3\left(\frac{\hat{q}_{BAP,h}BAP}{BAP+K_{BAP,h}}\right)\left(\frac{NaN}{K_{NaN,h}+NaN}\right)X_h$
18	Anoxic metabolism of BAP and NO_2^--N by X_h	γ_{NiN}		cY_p			$a_3\left(\frac{\hat{q}_{BAP,h}BAP}{BAP+K_{BAP,h}}\right)\left(\frac{NiN}{K_{NiN,h}+NiN}\right)X_h$
19	Metabolism of BAP by X_f						$a_3(a_1+a_2)\left(\frac{\hat{q}_{BAP,f}BAP}{BAP+K_{BAP,f}}\right)X_f$
20	Hydrolysis of PCOD						$k_P P$

	Process	Chemical components		Biomass components			Kinetic expression
		N_2	CH_4	X_h	X_a	X_n	
21	Aerobic respiration by X_h						$f_d b_h \left(\frac{DO}{K_{DO,h} + DO}\right) X_h$
22	Anoxic respiration by X_h during NO_3^--N utilization	$\gamma_{NaN}\ \gamma_O$					$f_d b_h a_3 \left(\frac{NiN}{K_{NiN,h} + NiN}\right) X_h$
23	Anoxic respiration by X_h during NO_2^--N utilization	$\gamma_{NiN}\ \gamma_O$					$f_d b_h a_3 \left(\frac{NaN}{K_{NaN,h} + NaN}\right) X_h$
24	Aerobic respiration by X_a						$f_d b_a \left(\frac{DO}{K_{DO,a} + DO}\right) X_a$
25	Aerobic respiration by X_n						$f_d b_n \left(\frac{DO}{K_{DO,n} + DO}\right) X_n$
26	Anaerobic respiration of X_f						$\gamma_O f_d b_f X_f a_3 (a_1 + a_2)$
27	Anaerobic respiration of X_m		1				$\gamma_O f_d b_m X_m a_3 (a_1 + a_2)$
28	Formation of BAP						$k_{hyd}EPS$
29	Decay of X_h			-1			$b_h X_h \left[\frac{DO}{K_{DO,h} + DO} + \frac{NaN}{K_{NaN,h} + NaN} + \frac{NiN}{K_{NiN,h} + NiN}\right]$
30	Decay of X_a				-1		$b_a \left(\frac{DO}{K_{DO,a} + DO}\right) X_a$
31	Decay of X_n					-1	$b_n \left(\frac{DO}{K_{DO,n} + DO}\right) X_n$
32	Decay of X_f						$b_f X_f a_3 (a_1 + a_2)$

	Process	Chemical components		Biomass components			Kinetic expression
		N_2	CH_4	X_h	X_a	X_n	
33	Decay of X_m		$f_d\gamma_O$				$b_m X_m a_3 (a_1 + a_2)$
34	UNITS	$\frac{\text{mgN}}{\text{L d}}$	$\frac{\text{mgCOD}}{\text{L d}}$	$\frac{\text{mgVSS}}{\text{L d}}$	$\frac{\text{mgVSS}}{\text{L d}}$	$\frac{\text{mgVSS}}{\text{L d}}$	

	Process	Biomass components			Kinetic expression
		X_f	X_m	X_i	
1	Aerobic metabolism of substrate by X_h				$\left(\frac{\hat{q}_S S}{K_{S,h} + S}\right)\left(\frac{DO}{K_{DO,h} + DO}\right)X_h$
2	Anoxic metabolism of substrate by X_h with NO_3^--N				$a_3\left(\frac{\hat{q}_{Nan} S}{K_{S,h} + S}\right)\left(\frac{NaN}{K_{NaN,h} + NaN}\right)X_h$
3	Anoxic metabolism of substrate by X_h with NO_2^--N				$a_3\left(\frac{\hat{q}_{Nin} S}{K_{S,h} + S}\right)\left(\frac{NiN}{K_{NiN,h} + NiN}\right)X_h$
4	Hydrolysis of inactive biomass				$a_3(a_1 + a_2)k_{hyd}$
5	Anaerobic utilization of substrate by X_f	cY_f			$a_3(a_1 + a_2)\left(\frac{\hat{q}_S S}{K_{S,f} + S}\right)X_f$
6	Aerobic metabolism of NO_2^--N by X_n				$\left(\frac{\hat{q}_n NiN}{K_{NiN,n} + NiN}\right)\left(\frac{DO}{K_{DO,n} + DO}\right)X_n$
7	Aerobic metabolism of NH_4^+-N by X_a				$\left(\frac{\hat{q}_a Amm}{K_{Amm} + Amm}\right)\left(\frac{DO}{K_{DO,a} + DO}\right)X_a$
8	Aerobic metabolism of acetate by X_h				$\left(\frac{\hat{q}_{Ac} Ac}{K_{Ac,h} + Ac}\right)\left(\frac{DO}{K_{DO,h} + DO}\right)X_h$
9	Anoxic metabolism of acetate by X_h with NO_3^--N				$a_3\left(\frac{\hat{q}_{Ac} Ac}{K_{Ac,h} + Ac}\right)\left(\frac{NaN}{K_{NaN,h} + NaN}\right)X_h$
10	Anoxic metabolism of acetate by X_h with NO_2^--N				$a_3\left(\frac{\hat{q}_{Ac} Ac}{K_{Ac,h} + Ac}\right)\left(\frac{NiN}{K_{NiN,h} + NiN}\right)X_h$
11	Acetate utilization by X_m		cY_m		$a_3(a_1 + a_2)\left(\frac{\hat{q}_{Ac,m} Ac}{K_{Ac,m} + Ac}\right)X_m$

	Process	Biomass components			Kinetic expression
		X_f	X_m	X_i	
12	Aerobic metabolism of UAP by X_h				$\left(\frac{\hat{q}_{UAP,h}UAP}{UAP+K_{UAP,h}}\right)\left(\frac{DO}{K_{DO,h}+DO}\right)X_h$
13	Anoxic metabolism of UAP and NO_3^--N by X_h				$a_3\left(\frac{\hat{q}_{UAP,h}UAP}{UAP+K_{UAP,h}}\right)\left(\frac{NaN}{K_{NaN,h}+NaN}\right)X_h$
14	Anoxic metabolism of UAP and NO_2^--N by X_h				$a_3\left(\frac{\hat{q}_{UAP,h}UAP}{UAP+K_{UAP,h}}\right)\left(\frac{NiN}{K_{NiN,h}+NiN}\right)X_h$
15	Metabolism of UAP by X_f	cY_p			$a_3(a_1+a_2)\left(\frac{\hat{q}_{UAP,f}UAP}{UAP+K_{UAP,f}}\right)X_f$
16	Aerobic metabolism of BAP by X_h				$\left(\frac{\hat{q}_{BAP,h}BAP}{BAP+K_{BAP,h}}\right)\left(\frac{DO}{K_{DO,h}+DO}\right)X_h$
17	Anoxic metabolism of BAP and NO_3^--N by X_h				$a_3\left(\frac{\hat{q}_{BAP,h}BAP}{BAP+K_{BAP,h}}\right)\left(\frac{NaN}{K_{NaN,h}+NaN}\right)X_h$
18	Anoxic metabolism of BAP and NO_2^--N by X_h				$a_3\left(\frac{\hat{q}_{BAP,h}BAP}{BAP+K_{BAP,h}}\right)\left(\frac{NiN}{K_{NiN,h}+NiN}\right)X_h$
19	Metabolism of BAP by X_f	cY_p			$a_3(a_1+a_2)\left(\frac{\hat{q}_{BAP,f}BAP}{BAP+K_{BAP,f}}\right)X_f$
20	Hydrolysis of PCOD				$k_P P$
21	Aerobic respiration by X_h				$f_d b_h\left(\frac{DO}{K_{DO,h}+DO}\right)X_h$
22	Anoxic respiration by X_h during NO_3^--N utilization				$f_d b_h a_3\left(\frac{NiN}{K_{NiN,h}+NiN}\right)X_h$

	Process	Biomass components			Kinetic expression
		X_f	X_m	X_i	
23	Anoxic respiration by X_h during NO_2^--N utilization				$f_d b_h a_3 \left(\frac{NaN}{K_{NaN,h} + NaN}\right) X_h$
24	Aerobic respiration by X_a				$f_d b_a \left(\frac{DO}{K_{DO,a} + DO}\right) X_a$
25	Aerobic respiration by X_n				$f_d b_n \left(\frac{DO}{K_{DO,n} + DO}\right) X_n$
26	Anaerobic respiration of X_f				$\gamma_O f_d b_f X_f a_3 (a_1 + a_2)$
27	Anaerobic respiration of X_m				$\gamma_O f_d b_m X_m a_3 (a_1 + a_2)$
28	Formation of BAP				$k_{hyd} EPS$
29	Decay of X_h			$1-f_d$	$b_h X_h \left[\frac{DO}{K_{DO,h} + DO} + \frac{NaN}{K_{NaN,h} + NaN} + \frac{NiN}{K_{NiN,h} + NiN}\right]$
30	Decay of X_a			$1-f_d$	$b_a \left(\frac{DO}{K_{DO,a} + DO}\right) X_a$
31	Decay of X_n			$1-f_d$	$b_n \left(\frac{DO}{K_{DO,n} + DO}\right) X_n$
32	Decay of X_f	-1		$1-f_d$	$b_f X_f a_3 (a_1 + a_2)$
33	Decay of X_m		-1	$1-f_d$	$b_m X_m a_3 (a_1 + a_2)$
34	UNITS	$\frac{mgVSS}{L\ d}$	$\frac{mgVSS}{L\ d}$	$\frac{mgVSS}{L\ d}$	

Inhibition terms for processes limited by nitrate, nitrite, and dissolved oxygen concentrations are defined as:

Nitrite: $\mathrm{a_1 = \frac{K_{s,NO_2}}{K_{s,NO_2}+NO_2}}$

Nitrate: $\mathrm{a_2 = \frac{K_{s,NO_3}}{K_{s,NO_3}+NO_3}}$

Dissolved oxygen: $\mathrm{a_3 = \frac{K_{s,DO}}{K_{s,DO}+DO}}$

Coefficients:

$\mathrm{c = 1 - k_{UAP} - k_{EPS}}$

γ_O = 160/113 mgCOD/mgVSS

γ_N = 14/113 mgN/mgVSS

γ_A = 64/60 mgCOD/mgAcetate

γ_{NiN} = 14/24 mgNO_2^--N/mgCOD

γ_{NaN} = 14/40 mgNO_3^--N/mgCOD

$$\alpha_a = 3.43 - (3.43Y_a\gamma_O + 1)\left[1 - k_{UAP} - k_{EPS}\left(1 + 3.43\frac{\gamma_N}{\gamma_O}\right)\right] - 3.43k_{UAP}$$

$$\alpha_n = 1.14(1 - Y_a\gamma_O)c$$

$$\alpha_h = (1 - k_{UAP} - k_{EPS})(1 - Y_h\gamma_O)$$

$$\alpha_{hp} = (1 - k_{UAP} - k_{EPS})(1 - Y_p\gamma_O)$$

Table A.4. Microorganisms' kinetic and stoichiometric parameters

				Heterotrophs	AOB	NOB	Fermenters	Methanogens
			Subscript	h	a	n	f	m
Kinetic Parameters		Symbol	Units					
True yield coefficient	Substrate	Y_j	mgVSS/mgCOD	0.45	0.33	0.083	0.2	0.077
	SMP	Y_p	mgVSS/mgCOD	0.5				
Maximum utilization rate	Substrate	$\hat{q}_j$	mgCOD/mgVSS-d	10	3.1	13	10	--
	UAP	$\hat{q}_{UAP,j}$	mgCOD/mgVSS-d	1.8	--	--	1.8	--
	BAP	$\hat{q}_{BAP,j}$	mgCOD/mgVSS-d	0.5	--	--	0.5	--
	Acetate	$\hat{q}_{Ac,j}$	mgAc/mgVSS-d	8.1	--	--	--	7
Half-maximum rate concentration	Substrate	$K_{S,j}$	mgCOD/L	10	1.5	2.7	10	--
	Acetate	$K_{Ac,j}$	mgAc/L	10	--	--	--	30
	DO	$K_{DO,j}$	mgDO/L	0.2	0.5	0.68	--	--
	UAP	$K_{UAP,j}$	mgCOD/L	100	--	--	100	--
	BAP	$K_{BAP,j}$	mgCOD/L	85	--	--	85	--
	NO_2^- or NO_3^-	$K_{N,j}$	mgN/L	0.2	1.5	2.7	--	--
UAP formation rate		k_{UAP}	mgCOD/mgCOD	0.05				
EPS formation rate		k_{EPS}	mgCOD/mgCOD	0.18				
Hydrolysis rate	EPS	k_{hyd}	1/d	0.17				
	PCOD	k_p	1/d	0.22				
Decay rate		$b_{,j}$	1/d	0.3	0.15	0.15	0.04	0.03
Fraction of biodegradable biomass		f_d	-	0.8				
				DO		NO_2^-		NO_3^-
Inhibition factor		$K_{s,l}$	mgDO/L	0.2		--		--
			mgN/L	--		0.2		0.2

j denotes the biomass subscript, l denotes the chemical species subscript.

Appendix A.4: References

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